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THE APPLICATION OF DIFFUSION APPROXIMATIONS TO THE STUDY OF TIM--ETC(U)
1978 J P LEHOCZKY

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Final Scientific Report
Grant AFOSR 74-2642

The Application of Diffusion
Approximations to the Study of
Time-Sharing Computers and
Transitory Queueing Systems

by

John P. Lehoczky
Principle Investigator

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The research carried out under grant AFOSR 74-2642 has been concentrated on two fundamental goals: the development of a new technique for analyzing stochastic systems, the diffusion approximation, and the application of diffusion approximations to the analysis of certain special stochastic models of Air Force interest. The specific models chosen for study were time-sharing computer systems, maintenance and repair systems, communication systems, and queueing networks in general.

The research carried out has shown that the technique of diffusion approximations provides an excellent tool for describing, analyzing, and optimizing stochastic models. It is especially important to note that the technique works well for very complex systems, systems which cannot be analyzed by ordinary methods. Several research papers have shown that often fairly simple closed form expressions for describing systems behavior can be derived using diffusion approximations, thus opening up the possibility of system optimization.

Several specific findings and accomplishments should be singled out as being especially noteworthy. First, an entirely new approach has been developed for describing and possibly for optimizing the scheduling of time sharing computer systems with multiple job types. This technique also will be of great importance in optimizing the scheduling of maintenance and repair facilities which must provide a variety of services to a variety of different equipment. Furthermore, in the area of maintenance and repair systems, methods for describing service centers consisting of specialized repair crews and analyzing the performance

of such systems were developed.

Second, a diffusion approximation for the general n-compartment model was developed. This model is of great importance in many applied areas: queueing networks, maintenance-repair systems, and biological modeling. This paper, published in December 1977, has in three months attracted worldwide attention and nearly 100 reprint requests.

Third, the method of diffusion approximations was applied to describe the performance and behavior of two other stochastic systems of Air Force interest: communication systems and personnel flow models. In the case of the communication system, the complicated phenomenon of "retry" behavior was modeled. With personnel flow models, a hierarchical organization (such as a branch of the U.S. military) was used. The effect of various hiring policies and retirement policies on the availability of various personnel at each level was discussed. This report is in preparation to be completed in June, 1978.

In summary, the research carried out under this grant has clearly shown the tremendous potential of the technique of diffusion approximations for analyzing stochastic models of Air Force interest. The approximations have proven to be highly accurate, relatively easy to develop, and often produce closed form expressions suitable for optimizing the system. Given its potential in the areas of repair-maintenance, communication, queueing network, and time-sharing computers, a study of its applicability to other areas of interest seems appropriate.

Appendix

Reports issued during grant period

Gaussian Approximations to Service Problems: A Communication System Example, (with D.P.Gaver). Published in Journal of Applied Probability, 13, 768-780 (1976).

Abstract: Messages arrive at a group of service channels in accordance with a time-dependent Poisson process. An arrival either (i) immediately begins k-stage Markovian service if an empty channel is reached, or (ii) balks and enters a retrial population if the channel sought is busy. Diffusion approximations to the number of messages in service (in each stage) and in the retrial population are derived by writing Ito stochastic differential equations. Steady state distributions are found and compared with certain simulation results.

A Diffusion Approximation Solution for a Repairman Problem with Two Types of Failure, (with D.P. Gaver). Published in Management Science, 24, 71-81 (1977).

Abstract: A new type of repairman model is proposed in which failures may require multiple repair operations, each done by a specialized repair crew. These repair operations may occur simultaneously, for example they may be carried out by specialized repair crews. An analysis of this model is carried out using diffusion approximation techniques. The analysis predicts that

the number of customers in service will have a steady state normal distribution with specified mean and covariance structure. Numerical studies are presented which substantiate this assertion in the infinite server case. An analysis for the multiple server case is also provided.

Formulas for Stopped Diffusion Processes with Stopping Times Based on the Maximum. Published in Annals of Probability, 5, 601-607 (1977).

Abstract: The joint Laplace transform of T and $X(T)$ is derived where $X(\cdot)$ is a time homogeneous diffusion process and T is the first time the process falls a specified amount below its current maximum. This generalizes the work of Taylor. The distribution of the maximum at T is shown to be exponential for Brownian motion. Formulas for more general stopping times based on the current maximum are given. The results have application to quick detection problems.

A Diffusion Approximation Analysis of a General n -Compartment System (with D.P. Gaver). Published in Mathematical Biosciences, 36, 127-148 (1977).

Abstract: A new approach to the stochastic analysis of general compartment models is presented. The analysis is based on the concept of diffusion approximations. The state of a compartment system is represented as the superposition of a deterministic process (characterized by a system of ordinary differential equations) and a

random noise process (characterized by stochastic differential equations). All transition-rate parameters are permitted to be time-dependent. Numerical solutions are presented for the two-compartment case and compared with those of Cardenas and Matis. Extensions to nonlinear compartment models are discussed.

Random Juror Selection from Multiple Lists (with J.B. Kadane)
published in Operations Research, 24, 207-219 (1976).

Abstract: The selection of jurors' names from multiple source lists is examined using statistical and optimization methodology. Five plans for sampling at random from overlapping lists of names are analyzed for their probabilistic and cost properties. In each plan the probability of a name being selected is independent of which and how many lists it appears on. We consider the optimal ordering of the lists to minimize cost and develop a heuristic for solving this problem. Although the methods are discussed in terms of juror selection, the results apply to sampling from overlapping lists in any context. For instance, if lists of equipment are kept according to possible uses, with versatile equipment listed many times, the methods of this paper can be used to draw a random sample of equipment to check for readiness.

Models for Work Backlogs at Computers that Time-Share Heterogeneous Users (with D.P. Gaver) Technical Report No. 132 Department of Statistics, Carnegie-Mellon University (Submitted for publication to Operations Research), 1977.

Abstract: Models of time-sharing computers with terminals which submit several different job types are introduced. Several approximation methods are introduced. Of primary importance is the introduction of a method which allows for performance evaluation as a function of the queue discipline. This opens up the possibility of the optimal scheduling of jobs. Diffusion approximations are also considered. The accuracy of all the approximation methods is assessed by numerical methods.

Approximate Models for Central Server Systems With Two Job Types (with D.P. Gaver) Technical Report No. 130 Department of Statistics, Carnegie-Mellon University, 1977 (submitted for publication to the Journal of the ACM).

Abstract: Two new approximation techniques for describing the performance of a closed queueing network when two or more job types are present are introduced and analyzed. The approximations apply in cases where a product form steady state solution can not be obtained, for example in the first-come-first-served single server queue case. This approximation provides a method of reducing the state space to a small fraction of that needed for an implementation of the Gaver-Humfeld method. Numerical results illustrate the excellent accuracy of these techniques.

A New Stochastic Analysis of Chemical Kinetics, Technical Report No. 129, Department of Statistics, Carnegie-Mellon University, 1977 (submitted to Journal of Chemical Physics).

Abstract: A new approach to the formulation and analysis of stochastic models of chemical reactions is presented. Unimolecular, biomolecular, and enzyme kinetic reactions are considered in the irreversible and reversible cases. The methodology is based on diffusion approximations and represents the time evolution of the reaction as the sum of a deterministic function and an Ornstein-Uhlenbeck process. As a result the marginal distributions are approximately Gaussian with relatively simple mean and covariance parameters, and the dynamic behavior is completely characterized. The stochastic approach which uses stochastic differential equations is a natural generalization of the deterministic approach which uses ordinary differential equations.

Manpower Models in a Hierarchical Organization (with D.P. Gaver), Technical Report in preparation, Department of Statistics, Carnegie-Mellon University.

Abstract: A model for personnel flow in a hierarchical organization is developed. The analysis is carried out by means of diffusion approximations. A variety of problems are addressed including the impact of preferential hiring policies and quotas. The transitory behavior and steady state behavior are both considered. Of particular importance is the availability of the workforce at all ranks under various recruitment and separation policies.

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